$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

$$\varepsilon^3 - I_1' \varepsilon^2 + I_2' \varepsilon - I_3' = 0$$

$$\varepsilon^3 = f_3(\varepsilon^0, \varepsilon^1, \varepsilon^2)$$
$$\varepsilon^4 = f_4(\varepsilon^0, \varepsilon^1, \varepsilon^2)$$

$$\cdots \quad \cdots$$

$$\varepsilon^n = f_n(\varepsilon^0, \varepsilon^1, \varepsilon^2)$$

$$\Rightarrow F = a_0 I + a_1 \varepsilon + a_2 \varepsilon^2 + a_3 \varepsilon^3 + \cdots$$

$$= A_0 I + A_1 \varepsilon + A \varepsilon^2$$

$$A_0 = h_0(I_1', I_2', I_3')$$
$$A_1 = h_1(I_1', I_2', I_3')$$
$$A_2 = h_2(I_1', I_2', I_3')$$

۳.۲

$$\sigma_{ji}\,\delta u_{i,j} = \frac{1}{۲}\sigma_{ji}\,\delta u_{i,j} + \frac{1}{۲}\sigma_{ji}\,\delta u_{i,j}$$

$$= \frac{1}{۲}\sigma_{ji}\,\delta u_{i,j} + \frac{1}{۲}\sigma_{ij}\,\delta u_{i,j}$$

$$= \frac{1}{۲}\sigma_{ji}\,\delta u_{i,j} + \frac{1}{۲}\sigma_{ji}\,\delta u_{j,i}$$

$$= \sigma_{ji}\,\frac{1}{۲}\left(\delta u_{i,j} + \delta u_{j,i}\right)$$

$$= \sigma_{ji}\,\delta \varepsilon_{ij}$$

۳۰۱۸

$$I'_1 = \varepsilon_{ii} \quad \& \quad I'_2 = \frac{1}{2}\varepsilon_{ij}\varepsilon_{ji}$$

$$\frac{\partial I'_1}{\varepsilon_{ij}} = \frac{\partial \varepsilon_{kk}}{\partial \varepsilon_{ij}} = \delta_{ki}\delta_{kj} = \boxed{\delta_{ij}} \qquad \text{۳.۲۷a}$$

$$\frac{\partial I'_2}{\varepsilon_{kl}} = \frac{1}{2}\frac{\partial \varepsilon_{ij}\varepsilon_{ji}}{\partial \varepsilon_{kl}} = \left(\varepsilon_{ji}\,\delta_{ki}\,\delta_{lj} + \varepsilon_{ij}\,\delta_{kj}\,\delta_{li}\right)\tfrac{1}{2}$$

$$\cancel{= \tfrac{1}{2}\left(\varepsilon_{ji}\delta_{ij} + \varepsilon_{ij}\delta_{ij}\right)}$$

$$= \frac{1}{2}\left(\varepsilon_{lk} + \varepsilon_{lk}\right) = \varepsilon_{lk} = \boxed{\varepsilon_{kl}} \qquad \text{۳.۲۷b}$$